***2.* System description**



**Fig. 1.** Diagram of the geothermal heat supply system

The geothermal heat supply system is presented in Fig. 1. Geothermal water is pumped from the geothermal well to drive the whole system. A filter is used to get rid of the impurity in the water. The geothermal water, at first, releases heat in a heat exchanger to drive the first heat supply cycle. Radiant heating system is employed to provide heat in the buildings because of the good comfort and low supply water temperature requirement. Water in the radiant floor heating cycle 1 absorbs heat in condenser 1 and then releases in the hop water pipe under the floor. After the condenser, the temperature of the geothermal water is about 40℃, which is much higher than the ambient temperature. To realize the cascading utilization of the geothermal water, a heat pump cycle is introduced. Working fluid in the heat pump cycle absorbs heat in the evaporator. The heated working fluid steam then is compressed by a compressor to a high-temperature and high-pressure state. The high-temperature and high-pressure working steam flows in a condenser to drive another radiant floor heating cycle. Water in the radiant floor heating cycle 2 absorbs heat in the condenser and then provide heat in the hot water pipe under the floor to heat the buildings.

Working fluid in the heat pump cycle is selected as R245fa for its good thermodynamic performance and low environment pollution effects.

**3. Mathematical models**

Before the analysis of the system, several assumptions are made to simplify the simulation of the system.

1. The system is in a steady state.
2. There is no fiction loss in the system pipe and no heat exchange between the system equipment and the ambient air.
3. The pressure losses in the evaporator and condensers are ignored.
4. The flow process in the throttle valve is isenthalpic.
5. The working fluid at the outlet of the evaporator is saturated vapor.

3.1. Thermodynamic analysis

3.1.1. System energy analysis

The energy analysis of the system complies with the conservation of mass and energy. Basic equations of the system are expressed as:

 (1)

 (2)

The equations for each component are listed in Table 1.

**Table 1.**

Equations for system components

|  |  |
| --- | --- |
| Component | Equation |
| Condenser 1 |  |
| Pump 1 |  |
| Evaporator |  |
| Compressor |  |
| Condenser 2 |  |
| Throttle valve |  |
| Pump 2 |  |

The heat supply by the radiant floor heating cycle 1 is expressed as:

 (3)

The heat supply by the radiant floor heating cycle 2 is expressed as:

 (4)

3.1.2. System exergy analysis

In general, the energy analysis of a system is based on the first law of the thermodynamic which focuses the amount of the thermal energy in the system. The exergy analysis of the system is based on the second law of the system which focuses on the quality of the thermal energy. The exergy analysis of the system is based on a dead state (the ambient conditions in this paper). The equation for the exergy flow rate in is expressed as:

 (5)

where *i* is the state point in the system and 0 is the state of ambient conditions

For each component, there is an exergy balance equation be expressed as: []

 (6)

where  is the exergy fuel rate and  is the exergy product rate of the system. The exergy fuel rate is equal to the sum of the exergy product rate, the exergy deduction and exergy loss. Thus,  donates the exergy deduction and  donates the exergy loss.

The exergy balance equations for each component are listed in Table 2.

**Table 2.**

Exergy analysis for each component for CHP system

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Component |  |  |  |  |
| Condenser 1 |  |  |  | / |
| Pump 1 |  |  |  | / |
| Radiant floor heating1 |  |  |  | / |
| Evaporator |  |  |  | / |
| Compressor |  |  |  | / |
| Condenser 2 |  |  |  | / |
| Throttle valve | / | / |  | / |
| Pump 2 |  |  |  | / |
| Radiant floor heating 2 |  |  |  | / |
| Reinjection | / | / | / |  |

The exergy efficiency of the system is given as:

 (7)

3.2. Heat supply analysis



**Fig. 2.** Diagram of the radiant floor heating cycle.

Radiant floor heating cycle is used in this geothermal heat supply system. Hot water pipe is lied under the floor to heat the room by radiation. Radiant-heat is more comfortable for human compared the traditional convection heat []. Moreover, the supply water temperature requirement is lower than traditional system, because of the large radiant heating area in the room. A suitable temperature range of the supply water is 30-60℃, causing the potential to utilize the low-grade heat. The diagram of the radiant floor heating cycle is shown in Fig. 3.

3.2.1. Heat loss in the building

The geothermal heating system operates in winter, when the ambient temperature is about -5℃. To keep the temperature in the building steady, heat supplied should be equal to the heat loss in the building under designed conditions. The heat loss of the building has two main components: heat loss through the build surroundings and air circulation between the ambient and the room. Other ways of the heat exchange of the room between the sunlight, human body, devices, etc. are not taken into consideration because of the small amount.

The heat loss through the wall can be calculated as:

 (8)

where  is the area of the surrounding wall.

 is the overall heat transfer coefficient, being expressed as:

 (9)

where  and  are the convection heat transfer coefficient of the inside area and the outside air, respectively.

 is the thermal conductivity of the material in the wall and  denotes the thickness of the material. The parameters of the wall are listed in **Appendix A**.

To keep the air in the room fresh, circulation of the room air is necessary. The equation of the air circulation is given as:

 (10)

where *V* is the amount of the circulation air and  is the concentration of the pollution outside the room;  is the pollution concentration inside the room and  is the amount of the pollution spreads inside the room.

The amount of the circulation air is

 (11)

The heat loss by the circulation of the room air is expressed as:

 (12)

3.2.2. Heat supply in the building

Heat is transferred to the room by means of radiation and convection. The equation of the radiation heat transfer is given by:

 (13)

where *F* is the configuration factor and is the Boltzmann’s constant.

is the average temperature of the floor surface, being given by:

 (14)

where  is the effective heat transferred to the building.

The heat transfer equation [] of convection has the following form

 (15)

where the value of and can be obtained in Table 3

**Table 3**

The value of *a* and *b*

|  |  |  |
| --- | --- | --- |
| condition | *a* | *b* |
| Heat transfer to the upper layer | 2.13 | 1.31 |
| Heat transfer to the lower layer | 0.14 | 1.25 |

As a result, the heat load required to keep the room temperature steady is obtained. The floor area for each room is known and the heat supply area is obtained.

3.3. The area of the heat exchanger

Tube and shell heat exchangers are used in this study for the high heat transfer efficiency and low capital price. There are two different ways of heat transfer in this study: the single-phase heat transfer process (liquid to liquid) and the two-phase heat transfer process (liquid to gas). The single-phase heat transfer process takes place in the condenser 1 while the two-phase heat transfer process takes place in the evaporator and condenser 2.

To simply the heat transfer process in the heat exchanger, the thermodynamic properties of the working fluid are assumed to be uniform and constant in a small section. Thus, the heat transfer process is discretized to a lot of subsections.

3.3.1. Single-phase heat transfer process

For each subsection, the heat transfer equation is given as:

** (16)

whereis the overall heat transfer coefficient in each subsection, **is the heat transfer area.

**is the log-mean temperature difference (LMTD) in the subsection, being expressed as:

** (17)

The overall heat transfer coefficient is expressed as:

** (18)

where  and  are the convection heat transfer coefficient of the hot fluid and the cold fluid, respectively; *λ* is the thickness of the tube and *δ* denotes the thermal conductivity of the tube wall.

In the tube side, the convection heat transfer coefficient is expressed as:

 (19)

The Nusselt number is expressed as: [] []

** (20)

** (21)

where the *f* is the Darcy frication factor []

 (22)

The Reynolds number can be expressed as:

** (23)**is the mass velocity of the fluid in the tube-side, being given by:

** (24)

where *N* is the number of the tubes inside the shell.

The Prandtl number can be calculated as:

** (25)

In the shell side, the convection heat transfer coefficient is expressed as: []

** (26)

where ** is the equivalent diameter in the shell side, being expressed as:

** (27)

*G*s is the mass velocity of the fluid in the shell side, being given by:

** (28)

where *A*mas is the maximum area in the flow, being determined as:

** (29)

3.3.2. Two-phase heat transfer process

Fluid in the two-phase heat transfer process has the vapor phase and the liquid phase. Heat transfer in the vapor and liquid mixture is different from that in the single-phase process. Likewise, the heat transfer region is discretized into many subsections, in which the thermodynamic properties of the mixture are assumed to be uniform and constant. Evaporation and condensation are two main two-phase heat transfer process.

In the condensation process, the heat transfer coefficient is given by:

** (30)

And in the evaporation process, the heat transfer coefficient is determined as:

** (31)

where ** is the boiling number.

** (32)

Subsequently, the heat transfer area in all the heat exchangers can be calculated by adding all the subsection heat transfer area together.

3.4. Costs and exergoeconomic analysis of the system

3.4.1. Cost analysis

To calculate the cost of the equipment in the system, the equipment module costing technique [] is employed in this study. At first, the bare module cost of the equipment is calculated as the basic cost. The base cost of the equipment includes the direct project cost (such as equipment cost, material cost for the installation, etc.) and the indirect cost (like the taxes, insurance engineering expenses etc.). The costing technique relates the costs back to the purchased cost of equipment evaluated for some base conditions. To solve this problem, some multiplying factors such as the specific equipment type, the specific system pressure and the specific material of construction are introduced.

Moreover, the base conditions are data depend on past records. Thus, the changing if the economic conditions such as the inflation should be taken into consideration. The equation of the correction is expressed as:

** (33)

where *C* is the purchased cost and *CEPCI* (Chemical Engineering Plant Cost Index) is the cost index; the values of *CEPCI*2018 and *CEPCI*ref,2001 is 638.1 and 397, respectively [46] [47].

The type of the heat exchanger used in this study is shell-and-tube and the material is carbon steel (CS). The cost of the heat exchanger considering the inflation is expressed as:

** (34)

where ** and ** is the constants determined by the type of the heat exchanger;** is the material factor and ** is the pressure factor.

For carbon steel as material, ** is 1. And the pressure factor is calculated from the following equation:

** (35)

where **, ** and ** are constants determined by the type of the heat exchanger.

The basic cost for the heat exchanger is expressed as:

** (36)

where ** is the constant determined by the type of the heat exchanger; *A* is the heat exchanger area.

The type of pumps used in this study are reciprocating and the material is stainless steel (SS). The cost of the pumps considering the inflation is expressed as:

** (37)

where ** and ** are the constants determined by the type of the pump; ** is the material factor and ** is the and pressure factor.

And the pressure factor is obtained from the following equation:

** (38)

where **, ** and ** are constants determined by the type of the pump.

The basic cost for the pump is given as:

** (39)

where ** is the constant determined by the type of the pump; **is the power consumption of the pump.

The type of the compressor used in this study is axial type and the material is carbon steel (CS). The cost of the compressor considering the inflation is given as:

** (40)

where ** is the module factor of the compressor.

The basic cost for the compressor is given as:

** (41)

where ** is the constant determined by the type of the compressor; ** is the power consumption of the compressor.

The values of the constants mentioned above for the main components are listed **Appendix B.**

3.4.2. Exergoeconomic analysis

Exergoeconomic is a branch of engineering which combines the thermodynamic analysis and economic principles. Thermodynamic performance and economic cost of the system are all taken into consideration. In the exergoeconomic analysis, an average cost per unit of exergy. is associated with each exergy stream.

The capital recovery factor (CRF) is employed to find the relationship between the present value of the expenditure and the equivalent annually levelized costs. The equation of the relationship is given as:

** (42)

** (43)

where ** is the effective discount rate and n is the lifetime of the geothermal heat supply system; in this study, ** is given as 5% [49] and *n* is assumed as 30 [50].

In this study, the annual working time is the system is assumed to be 8000h in order to calculate the equivalent annually levelized costs.

In a steady system, there are a number of entering and outing working fluid steams and there are heat and work interactions with the surroundings. As mentioned before, each flowing steam is associated with an average cost per unit of exergy. The equations to calculate the cost of the steam product are given as:

** (44)

** (45)

** (46)

** (47)

where **, **, ** and ** denote average costs per unit of exergy of the steams; and are the exergy transfer rate of the steam flowing in and out a component;  and are the power and the heat transfer rate of the component considering the annual working time.

The equation of the cost balance for each component is given as:

** (48)

Details of the cost balance for each component are listed in Table 4.

**Table 4.**

Cost balance for components

|  |  |  |
| --- | --- | --- |
| Component | Cost balance | Auxiliary relation |
| Geothermal resource |  | / |
| Condenser 1 |  |  |
| Pump 1 |  | / |
| Floor heating pipe 1 |  |  |
| Evaporator |  |  |
| Compressor |  | / |
| Condenser 2 |  |  |
| Throttle valve | / |  |
| Pump 2 |  | / |
| Floor heating pipe 2 |  |  |

Heat supplied by the system is the product of the system. Average cost per unit of exergy for the system product is chose to represent the exergoeconomic performance, being expressed as: [52] [53]

 (49)

**4. Results and discussion**

The thermodynamic properties of the working fluids in the geothermal heat supply system are obtained from REFPROP 9.1 a software developed by the National Institution of the Standards and Technology of the United States. Program used to simulate the system are written in MATLAB. The basic parameters for the simulation are listed in Table 5.

**Table 5**

Basic parameters for simulation

|  |  |
| --- | --- |
| Term | Value |
| Geothermal water temperature (℃) | 85 |
| Geothermal water pressure (kPa) | 200 |
| Mass flow of geothermal water (kg/s) | 36.11 |
| Ambient temperature (℃) | 5 |
| Ambient pressure (kPa) | 101.3 |
| Isentropic efficiency of pump 1 (%) | 75 |
| Isentropic efficiency of pump 2 (%) | 75 |
| Isentropic efficiency of the compressor (%) | 80 |
| Geothermal water temperature at the outlet of condenser1 (℃) | 30 |
| Geothermal water temperature at the outlet of evaporator (℃) | 18 |
| Pressure at the inlet of the compressor (kPa) | 80 |
| Compressor pressure ratio | 6 |
| Inlet temperature of radiant floor heating supply water (℃) | 45 |
| Outlet temperature of radiant floor heating supply water (℃) | 35 |
| Heat load per area (kW/m2) | 0.07 |

4.1. Thermodynamic and exergoeconomic analysis of the system

Three key parameters (geothermal water temperature at the evaporator inlet, reinjection geothermal water temperature, compressor pressure ratio) are chosen to analyze the thermodynamic and exergoeconomic performance of the system. In the thermodynamic aspect, the heat load of the first radiant floor heating cycle (*Q*radiant floor heating 1), heat load of the whole system (*Q*radiant floor heating 1 & 2), compressor consumption (*W* compressor) and the exergy efficiency of the system (**) are selected. Average cost per unit of exergy for the system heat load (**), average cost per unit of exergy for the heat load by the heat pump part (**) and the total cost of the system (*Z*cost) are chosen to reflect the exergoeconomic part.



**Fig. 3.** Effects of the geothermal water temperature at the evaporator inlet on the thermodynamic performance of the system.

Fig. 3 shows the effects of the geothermal water temperature at the evaporator on the thermodynamic performance of the system. The inlet of the evaporator is the same place of the outlet of condenser 1. As the geothermal water temperature at the outlet of condenser 1 increases, less heat in the geothermal water is used in condenser 1. Thus, the heat load provided by the radiant floor heating cycle decreases.

With the increase of the geothermal water temperature at the evaporator inlet, more heat is provided in the evaporator. The mass flow rate of the working fluid in the heat pump cycle increases, resulting in the increase of the head load in radiant floor heating cycle 2. The increase of the heat load in cycle 2 is more than that in cycle 1. Thus, the total heat load of the geothermal water heat supply system increases, despite the decrease of the heat load in the radiant floor heating cycle 1.

The exergy efficiency of the system decreases as shown in Fig. 3. The increase of the geothermal water temperature at the evaporator inlet causes the increase of the mass flow rate of the working fluid in the heat pump cycle. As a result, more power is consumed by the compressor when compressing the vapor. Exergy consumed by the compressor is more than the exergy produced in the evaporator. Thus, though more heat load is produced by the heat supply system, the exergy efficiency of the system decreases on the contrary.



**Fig. 4.** Effects of the geothermal water temperature at the evaporator inlet on the exergoeconomic performance of the system.

Fig. 4 presents the effects of the geothermal water temperature at the evaporator inlet on the exergoeconomic performance of the system. As the increase of the geothermal water temperature at the evaporator inlet, the heat transfer area needed in condenser 1 decreases, cutting down the cost of the condenser 1. However, the increase of the mass flow rate of the working fluid rise the cost of all the components in the heat pump cycle. The heat transfer area in the evaporator and condenser 2 increases. Moreover, the power consumption of the compressor increases, leading to the increase of the cost of the compressor. As a result, the total cost of the system increases.

The heat load provided by the heat pump is large. The increase of the heat load produced by the heat pump is faster than the increase of the total system cost. Thus, the average cost per unit of exergy for the heat pump product (heat load produced by radiant floor heating cycle 2) decreases.

Counting the decrease of the heat load in the radiant floor heating cycle 1, the increase of the heat load by the whole system can’t catch the increase of the total system cost. As a result, the average cost per unit of exergy for the system product decreases.



**Fig. 5.** Effects of the reinjection geothermal water temperature on the thermodynamic performance of the system.

Fig. 5 shows the effects of the reinjection geothermal water temperature on the thermodynamic performance of the system. The increase of the reinjection geothermal water temperature can’t affect the performance of the radiant heating system 1. Thus, the heat load of the first radiant floor heating cycle remains unchanged.

The increase of the reinjection geothermal water temperature means the decrease of the heat provided in the evaporator by the geothermal water. Thus, the mass flow rate of the working fluid in the heat pump cycle decreases, resulting in the decrease of the heat load provided by the radiant floor heating cycle 2. Thus, the heat load of the whole system decreases.

The decrease of the working fluid mass flow rate causes the decrease of the power consumption of the compressor. Thus, the exergy consumed be the compressor decreases, causing the increase of the exergy efficiency of the system.



**Fig. 6.** Effects of the reinjection geothermal water temperature on the exergoeconomic performance of the system.

Fig. 6 presents the effects of the reinjection geothermal water temperature on the exergoeconomic performance of the system. The heat transfer area in condenser 1 remains unchanged. The decrease of the mass flow rate in the heat pump cycle cuts the cost of the evaporator, condenser and the compressor. Thus, the total cost of the system decreases.

The decrease of the heat load provided by the radiant floor heating cycle 2 is faster than the decrease of the cost of the equipment. Thus, the average cost per unit of exergy for the heat pump heat load increases. The average cost per unit of exergy for the heat load provided by the radiant floor heating cycle 1 is smaller than that of the geothermal heat supply system while the average cost per unit of exergy for the heat pump heat load is larger than that of the system. The average cost per unit of exergy for the system heat load consists of the average cost per unit of exergy for the radiant floor heating cycle 1 and 2. Thus, the increase of the reinjection geothermal water temperature reduces the occupation of the second cycle part. Thus, the average cost per unit of exergy for the system heat load approaches the average cost per unit of exergy for the radiant floor heating cycle 1 and becomes smaller.



**Fig. 7.** Effects of the compressor pressure ratio on the thermodynamic performance of the system.

Fig. 7 shows the effects of the compressor ratio on the thermodynamic performance of the system. The compressor pressure ratio can’t affect the thermodynamic performance of the radiant floor heating cycle 1. Thus, the heat load of the radiant floor heating cycle remains unchanged. The increase of the compressor pressure ratio causes the increase of the power consumption of the compressor. At the same time, the temperature and pressure of the working fluid vapor increases, causing the increase of the enthalpy of the vapor. The compressor provides energy for the vapor. As a result, more heat is provided in the condenser 2, leading to the increase of the heat load in the radiant floor heating cycle. Also, the increase of the compressor consumption results in the decrease of the exergy efficiency of the system.



**Fig. 8.** Effects of the compressor pressure ratio on the exergoeconomic performance of the system.

Fig. 8 presents the effects of the compressor pressure ratio on the exergoeconomic performance of the system. The increase of the pressure ratio causes the increase of the compressor consumption, requiring the increase of the compressor size. Thus, the cost of the compressor increases. The rise of the vapor enthalpy at the inlet of condenser 2 causes the increase of the heat transfer area of the condenser 2, raising the cost of the condenser 2. As a result, the total cost of the system increases. The increase of the cost of the compressor is faster than the increase of the heat load in the radiant floor heating cycle 2. Thus, the average cost per unit of exergy for the heat pump cycle increases. The cost of the compressor takes up a large part of the total cost and the increase of it is faster than the increase of the total heat load. Thud, the average cost per unit of exergy for the system heat load increases.

4.2. System comparing



**Fig. 9.** Diagram of the basic geothermal heat supply system.

The parameters analysis of the system shows that the improve of the geothermal water temperature at the evaporator inlet rises the heat load of the system while also causes the increase of the average cost per unit of exergy for the system product. On the contrary, the increase of the reinjection geothermal water temperature causes the decrease of the system heat load and the average cost per unit of exergy for the system product.